

elsewhere in order to represent the position of the crankshaft [1] and [11]. Such function has a high level of correlation with the measured in-cylinder pressure or with the measured indicated torque since it effectively represents the motored pressure or motored torque information. For the crankshaft speed signal, the relevant signal is the crankshaft velocity signal fluctuating around its mean value. Therefore, the general correlation function for estimating the in-cylinder pressure or indicated torque can be written as a function of the position function f_θ , angular speed fluctuation $\dot{\theta}$, and angular acceleration $\ddot{\theta}$, as shown below.

$$\text{Estimated Value} = F(f_\theta, \dot{\theta}, \ddot{\theta}) \quad (5)$$

ESTIMATION OF IN-CYLINDER PRESSURE

After the in-cylinder combustion pressure is estimated based on the crankshaft speed measurement, the indicated torque is then calculated accordingly based on the estimated in-cylinder pressure and the given engine geometry. The estimation model function (*basis function*) may be set to be the following first-order non-linear model as shown in Eq. (6) in order to first estimate the in-cylinder pressure.

$$P_{\text{estimate}} = a_0 + a_1 f_\theta + a_2 f_\theta \dot{\theta} + a_3 f_\theta \ddot{\theta} + a_4 \dot{\theta} \ddot{\theta} \quad (6)$$

The stochastic estimation approach requires building the cross-correlation functions between the estimation quantity (in-cylinder pressure) and the measured quantities (three basic variables as well as their cross-terms as shown in Eq. (6)). The coefficients, a_0 through a_4 , can be obtained by minimizing the mean square difference between the measured pressure and the estimated pressure as shown in Eq. (7).

$$\varepsilon = \min_{a_i} \left(\sum_{k=1}^N (P_{\text{measured},k} - P_{\text{estimate},k})^2 \right) \quad (7)$$

As described earlier in Eqs. (2) and (3), taking the partial derivatives with respect to each of the coefficients and setting the result equal to zero gives the following cross-correlation matrix system to solve.

$$\begin{bmatrix} \langle 1 \rangle & \langle f_\theta \rangle & \langle f_\theta \tilde{\theta} \rangle & \langle f_\theta \tilde{\theta}' \rangle & \langle \tilde{\theta} \tilde{\theta} \rangle \\ \langle f_\theta \rangle & \langle f_\theta^2 \rangle & \langle f_\theta^2 \tilde{\theta} \rangle & \langle f_\theta^2 \tilde{\theta}' \rangle & \langle f_\theta \tilde{\theta} \tilde{\theta} \rangle \\ \langle f_\theta \tilde{\theta} \rangle & \langle f_\theta^2 \tilde{\theta} \rangle & \langle f_\theta^2 \tilde{\theta}^2 \rangle & \langle f_\theta^2 \tilde{\theta} \tilde{\theta}' \rangle & \langle f_\theta \tilde{\theta}' \tilde{\theta} \rangle \\ \langle f_\theta \tilde{\theta}' \rangle & \langle f_\theta^2 \tilde{\theta}' \rangle & \langle f_\theta^2 \tilde{\theta} \tilde{\theta}' \rangle & \langle f_\theta^2 \tilde{\theta}'^2 \rangle & \langle f_\theta \tilde{\theta} \tilde{\theta}' \rangle \\ \langle \tilde{\theta} \tilde{\theta} \rangle & \langle f_\theta \tilde{\theta} \tilde{\theta} \rangle & \langle f_\theta \tilde{\theta}' \tilde{\theta} \rangle & \langle f_\theta \tilde{\theta} \tilde{\theta}' \rangle & \langle \tilde{\theta}^2 \tilde{\theta}' \rangle \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \langle P \rangle \\ \langle P f_\theta \rangle \\ \langle P f_\theta \tilde{\theta} \rangle \\ \langle P f_\theta \tilde{\theta}' \rangle \\ \langle P \tilde{\theta} \tilde{\theta} \rangle \end{bmatrix} \quad (8)$$

In Eq. (8), the various terms in the matrix represent the cross-correlations among the measured basis variables while the right side of the equation represents the cross-correlations between the measured in-cylinder pressure and the measured basis variables. These non-linear cross-correlations are pre-computed based on all available data at a certain engine operating condition, then the five coefficients are computed once for all (cycles and cylinders) at that operating point. Once the coefficients as well as these correlation functions are determined and proper processing has been carried out, the estimation procedure reduces down to the simple evaluation of a multivariate polynomial form based on the measurements. Therefore, during the estimation phase the instantaneous value of the five measured basis variables are used to evaluate the simple polynomial equation as shown in Eq. (6) for the desired estimation. Therefore, the computational requirements can become very minimal in this approach, and the estimation can be achieved in real time with a few computational operations.

Referring to Fig. (2), Fig. (2) represents each of the prescribed basis variables including the in-cylinder combustion pressure position function f_θ . Based on these variables, the in-cylinder pressure was estimated using the basis function described in Eq. (6) and the cross-correlation described in Eq. (8). Referring to Fig. (3), Fig. (3)

$$\text{Normalized R.M.S. Error} = \frac{\left\langle \sqrt{\frac{1}{N} \sum_{i=1}^N (p_{est,i} - p_{meas,i})^2} \right\rangle}{\langle p_{meas} \rangle} \quad (9)$$

Table (4) illustrates this estimation error for each of the estimations and number of resolutions accounted in the computation. Note that the values are averages over all engine operating conditions.

Table 4. Normalized R.M.S. Errors for Various Cases

Estimation Type		Number of Resolutions		
		360	60	36
Indicated Pressure		2.694 %	5.063 %	3.494 %
Indicated Torque	Individual Cylinder	3.394 %	5.810 %	4.313 %
	All Cylinder	6.159 %	7.603 %	6.814 %

ESTIMATION OF INDICATED TORQUE

The indicated torque is estimated directly from the crankshaft speed measurements, replacing the two steps procedure of first estimating the in-cylinder pressure and secondly calculating the indicated torque accordingly. There are two different parts of achieving the indicated torque estimation in this approach. The first part is to estimate the individual cylinder torque for each cylinder then calculate their summations whereas the other part is to directly estimate the summation of individual cylinder torque.

Basis Function Selection - Various basis functions are investigated in order to determine the best form of the estimation model for the indicated torque estimation in real-time.

Table 5. Various Basis Functions

Function Number	Basis Function
1	$T_{estimate} = a_0 + a_1 f_\theta + a_2 \hat{\theta} + a_3 \hat{\theta}^2$

2	$T_{estimated} = a_0 + a_1 f_\theta + a_2 \tilde{\theta} + a_3 \tilde{\theta}^2 + a_4 \tilde{\theta}^3$
3	$T_{estimated} = a_0 + a_1 f_\theta + a_2 f_\theta \tilde{\theta} + a_3 f_\theta \tilde{\theta}^2 + a_4 \tilde{\theta} \tilde{\theta}^2$
4	$T_{estimated} = a_0 + a_1 f_\theta + a_2 \tilde{\theta} + a_3 \tilde{\theta}^2 + a_4 f_\theta \tilde{\theta} + a_5 f_\theta \tilde{\theta}^2 + a_6 \tilde{\theta} \tilde{\theta}^2$
5	$T_{estimated} = a_0 + a_1 f_\theta + a_2 \tilde{\theta} + a_3 \tilde{\theta}^2 + a_4 f_\theta^2 + a_5 \tilde{\theta}^3 + a_6 \tilde{\theta}^4$
6	$T_{estimated} = a_0 + a_1 f_\theta + a_2 f_\theta \tilde{\theta} + a_3 f_\theta \tilde{\theta}^2 + a_4 \tilde{\theta}^3 + a_5 \tilde{\theta} \tilde{\theta}^2 + a_6 \tilde{\theta}^3$
7	$T_{estimated} = a_0 + a_1 f_\theta + a_2 \tilde{\theta} + a_3 \tilde{\theta}^2 + a_4 f_\theta^2 + a_5 f_\theta \tilde{\theta} + a_6 f_\theta \tilde{\theta}^2 + a_7 \tilde{\theta}^3 + a_8 \tilde{\theta} \tilde{\theta}^2 + a_9 \tilde{\theta}^3$

Considering the estimation accuracy, number of terms, equation order, variable selection, etc., several different forms of basis functions were investigated using the different resolutions (36, 60, and 360) and all engine operating conditions. Table (5) describes each of the basis functions selected from many basis functions that were examined.

Note here that the position function f_θ for estimating the indicated torque is different from the previous one used for the in-cylinder pressure estimation. It is effectively a normalized motored torque, which can be calculated from the given engine geometry, for each individual cylinder as well as summation of all cylinders.

Coefficient Training – After selecting one of the prescribed basis functions in Table (5), the polynomial coefficients were obtained by taking the same procedures, as described in Eqs. (7) and (8). Then, the instantaneous value of the measured basis variables or their combinations were used to evaluate each of the polynomial equations shown in Table (5) to estimated the desired indicated torque. For instance, choosing the basis function 3 would result in the following cross-correlation matrix system.

$$\begin{bmatrix} \langle 1 \rangle & \langle f_\theta \rangle & \langle f_\theta \tilde{\theta} \rangle & \langle f_\theta \tilde{\theta}^2 \rangle & \langle \tilde{\theta} \tilde{\theta}^2 \rangle \\ \langle f_\theta \rangle & \langle f_\theta^2 \rangle & \langle f_\theta^2 \tilde{\theta} \rangle & \langle f_\theta^2 \tilde{\theta}^2 \rangle & \langle f_\theta \tilde{\theta} \tilde{\theta}^2 \rangle \\ \langle f_\theta \tilde{\theta} \rangle & \langle f_\theta^2 \tilde{\theta} \rangle & \langle f_\theta^3 \tilde{\theta} \rangle & \langle f_\theta^3 \tilde{\theta}^2 \rangle & \langle f_\theta \tilde{\theta}^3 \rangle \\ \langle f_\theta \tilde{\theta}^2 \rangle & \langle f_\theta^2 \tilde{\theta}^2 \rangle & \langle f_\theta^3 \tilde{\theta}^2 \rangle & \langle f_\theta^4 \tilde{\theta}^2 \rangle & \langle f_\theta \tilde{\theta}^4 \rangle \\ \langle \tilde{\theta} \tilde{\theta}^2 \rangle & \langle f_\theta \tilde{\theta} \tilde{\theta}^2 \rangle & \langle f_\theta^2 \tilde{\theta} \tilde{\theta}^2 \rangle & \langle f_\theta^3 \tilde{\theta} \tilde{\theta}^2 \rangle & \langle \tilde{\theta}^4 \tilde{\theta}^2 \rangle \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \langle T \rangle \\ \langle T f_\theta \rangle \\ \langle T f_\theta \tilde{\theta} \rangle \\ \langle T f_\theta \tilde{\theta}^2 \rangle \\ \langle T \tilde{\theta} \tilde{\theta}^2 \rangle \end{bmatrix} \quad (10)$$